Advanced Math

6-4

Vectors and Dot Products

Vector Operations - Given $\boldsymbol{u} = \langle a, b \rangle$ and $\boldsymbol{v} = \langle c, d \rangle$

Vector addition: $\boldsymbol{u} + \boldsymbol{v} =$ $\langle a+c, b+d \rangle$

answer is scalar / vector

Scalar Multiplication : $k \cdot \boldsymbol{v} =$ $\langle kc, kd \rangle$

answer is scalar / vector

Dot Product : $\boldsymbol{u} \cdot \boldsymbol{v} =$ $ac+bd$

answer is scalar / vector

$\vec{u} \times \vec{v}$

Do vectors exhibit the field following properties?

Given $\boldsymbol{u} = \langle a, b \rangle$ and $\boldsymbol{v} = \langle c, d \rangle$ and $\boldsymbol{w} = \langle x, y \rangle$

Commutative property; is $\boldsymbol{u} + \boldsymbol{v} = \boldsymbol{v} + \boldsymbol{u}$? Yes.

$\langle a+c, b+d \rangle = \langle c+a, d+b \rangle$

Commutative property; is $\boldsymbol{u} \cdot \boldsymbol{v} = \boldsymbol{v} \cdot \boldsymbol{u}$? Yes

$ac+bd = ca+db$

Distributive property; is $\boldsymbol{u} \cdot (\boldsymbol{v} + \boldsymbol{w}) = \boldsymbol{u} \cdot \boldsymbol{v} + \boldsymbol{u} \cdot \boldsymbol{w}$? Yes

$\langle a,b \rangle \cdot \langle c+x, d+y \rangle = ac+bd+ax+by$

$a(c+x) + b(d+y) =$

$ac+ax+bd+by = ac+ax+bd+by$ □

Vectors also have the following properties.

$\vec{v} = \langle a,b \rangle$

$\|\vec{v}\| = \sqrt{a^2+b^2}$, squared: 2

1) Dot by zero: $\mathbf{0} \cdot \boldsymbol{v} = 0$

answer is scalar / vector

2) Vector Dot Squaring: $\boldsymbol{v} \cdot \boldsymbol{v} = \|\boldsymbol{v}\|^2$

$\langle a,b \rangle \cdot \langle a,b \rangle = a \cdot a + b \cdot b = a^2+b^2$

answer is scalar / vector

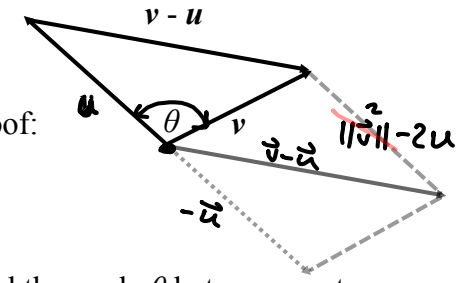
3) Scalar and Dot multiplication: $c(\boldsymbol{u} \cdot \boldsymbol{v}) = c\boldsymbol{u} \cdot \boldsymbol{v} = \boldsymbol{u} \cdot c\boldsymbol{v}$

answer is scalar / vector

Angle between two vectors: $\cos\theta = \dfrac{u \cdot v}{\|u\|\,\|v\|}$

Use Law of Cosines.

Proof:



$\|\vec{v}-\vec{u}\|^2 = \|\vec{u}\|^2 + \|\vec{v}\|^2 - 2\|\vec{u}\|\|\vec{v}\|\cos\theta$

FOIL

$\|\vec{v}\|^2 - 2u\cdot v + \|\vec{u}\|^2 = \|\vec{u}\|^2 + \|\vec{v}\|^2 - 2\|\vec{u}\|\|\vec{v}\|\cos\theta$

$\dfrac{-2u\cdot v}{-2\|\vec{u}\|\|\vec{v}\|} = \cos\theta$

Find the angle θ between vectors.

*) $u = 2i - 5j$
$v = -i + 7j$

$\vec{u}\cdot\vec{v} = -2-35 = -37$

$\|\vec{u}\| = \sqrt{29}$

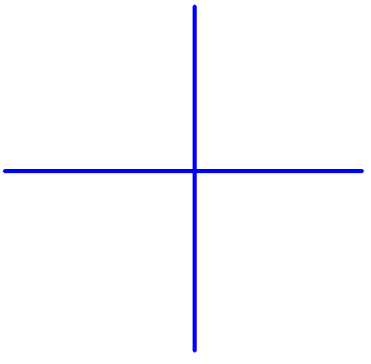
$\|\vec{v}\| = \sqrt{50}$

$\cos\theta = \dfrac{-37}{\sqrt{29}\cdot\sqrt{50}}$

$\theta = \cos^{-1}(-.97167)$

$\theta = 166.329°$

Orthogonal Vectors - Two vectors u and v are orthogonal (angle between vectors is 90°) if $u \cdot v = 0$



Assignment:
pg. 553
2-34 even.

kilogram